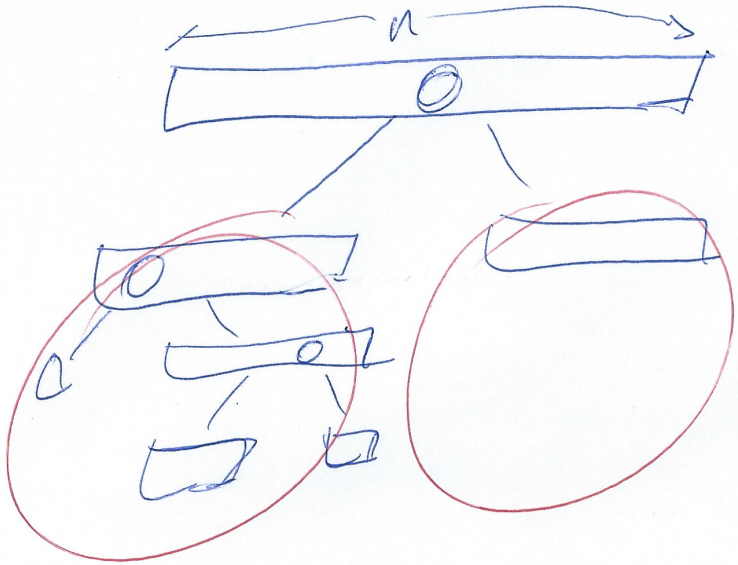
(1)

RANDOMIZED QUICK SORT



DEPTH $\approx \log n$

COUNT COMPARISONS

$a :: b$

why are a, b compared ?

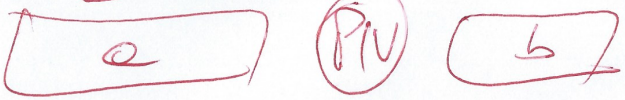
NOT

why a is compared against b ?

IF AND ONLY IF a or b is CHOSEN AS A PIVOT BEFORE ANY ELEMENT $a < x < b$

$a < \square\square\square - a\ b$ SORTED

SOME PIVOT IS FOUND BEFORE a OR b IS A PIVOT



EXPECTED # OF COMPARISONS = $\{1, 2, 3, \ldots, n\}$ SET TO BE SORTED

$$= \sum_{i=1}^{n} \sum_{J=1}^{n-1} p_{ij} \in \Theta(n \lg n)$$

$J - i + 1$ VALUES

p_{ij} = PROBABILITY OF COMPARING i VS. J = PROB. THAT IN $[i, \ldots, J]$ THE FIRST PIVOT WE CHOOSE IS EITHER i OR J

$$\sum_{i}^{n} \sum_{J}^{n} \frac{2}{(J-i)+1} \equiv \sum_{k=1}^{n} \sum_{z=1}^{n} \frac{2}{z}$$

$\frac{2}{J-i+1}$

HARMONIC SERIES

$$\sum_{z=1}^{n} z \sim \log_e n + 0.566\ldots$$

γ

WHY NOT PREFERRING QUICK SORT

- NOT STABLE

STABLE SORTING
DOES NOT EXCANGE ELEMENT
HAVING THE SAME VALUE

MERGE SORT → STABLE

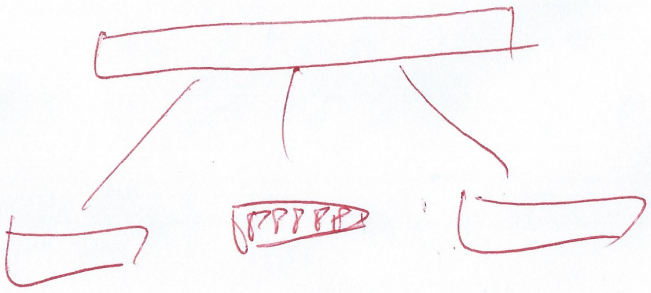
FEW DISTINCT

- MANY ELEMENTS, FEW VALUES ⇝ $O(n^2)$

10^6 ELEMENTS
IN $\{1, 20\}$

→ THREE WAYS QUICK SORT

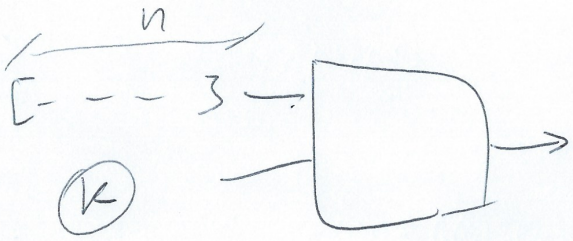
- SMALLER THAN PIVOT
- = PIVOT
- LARGER THAN PIVOT



④ SELECTION PROBLEM (K-th ELEMENT PROBLEM)

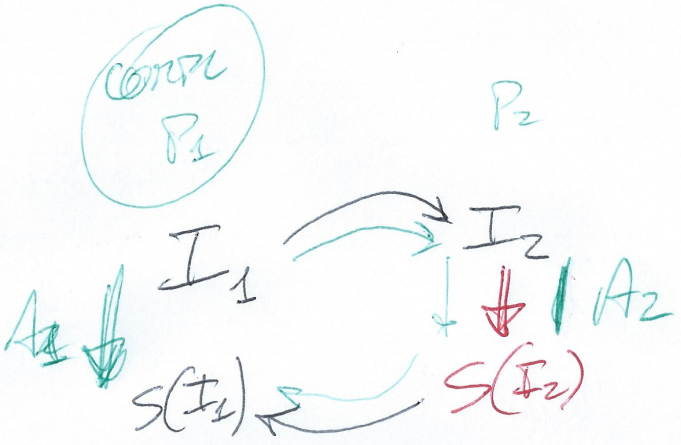
- FIND MINIMUM (1-st ELEM)
- FIND MAXIMUM (n-th ELEM)
- FIND MEDIAN ($\frac{n}{2}$-th ELEM)

CAN BE BY SORTING → $O(n \log n)$



ELEMENT WITH RANK = K

ELEMENT THAT IS IN K-POSITION WHEN SORTED



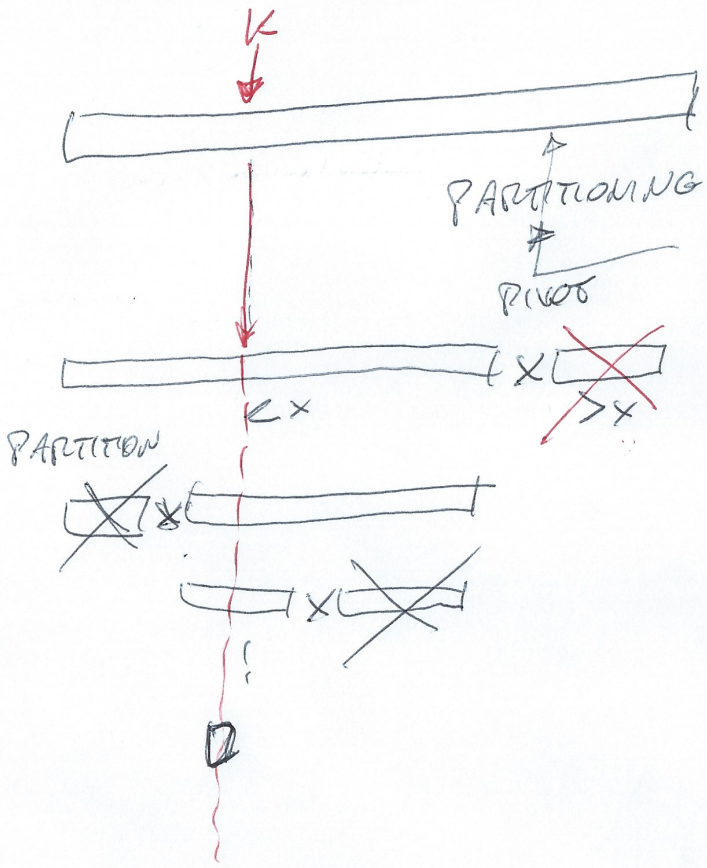
SELECTION PROBLEM NOT MORE DIFFICULT THAN SORTING

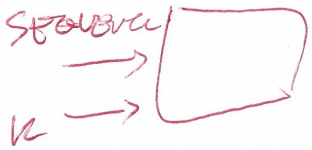
REDUCTION FROM P_1 TO P_2

ALGORITHM FOR P_2 ALSO SOLVES P_1

9 SELECTION K-th elem. IN $\Theta(n)$ EXPECTED TIME

"PARTIAL" QUICK SORT





START $\leq K \leq$ END

~~def SEL~~

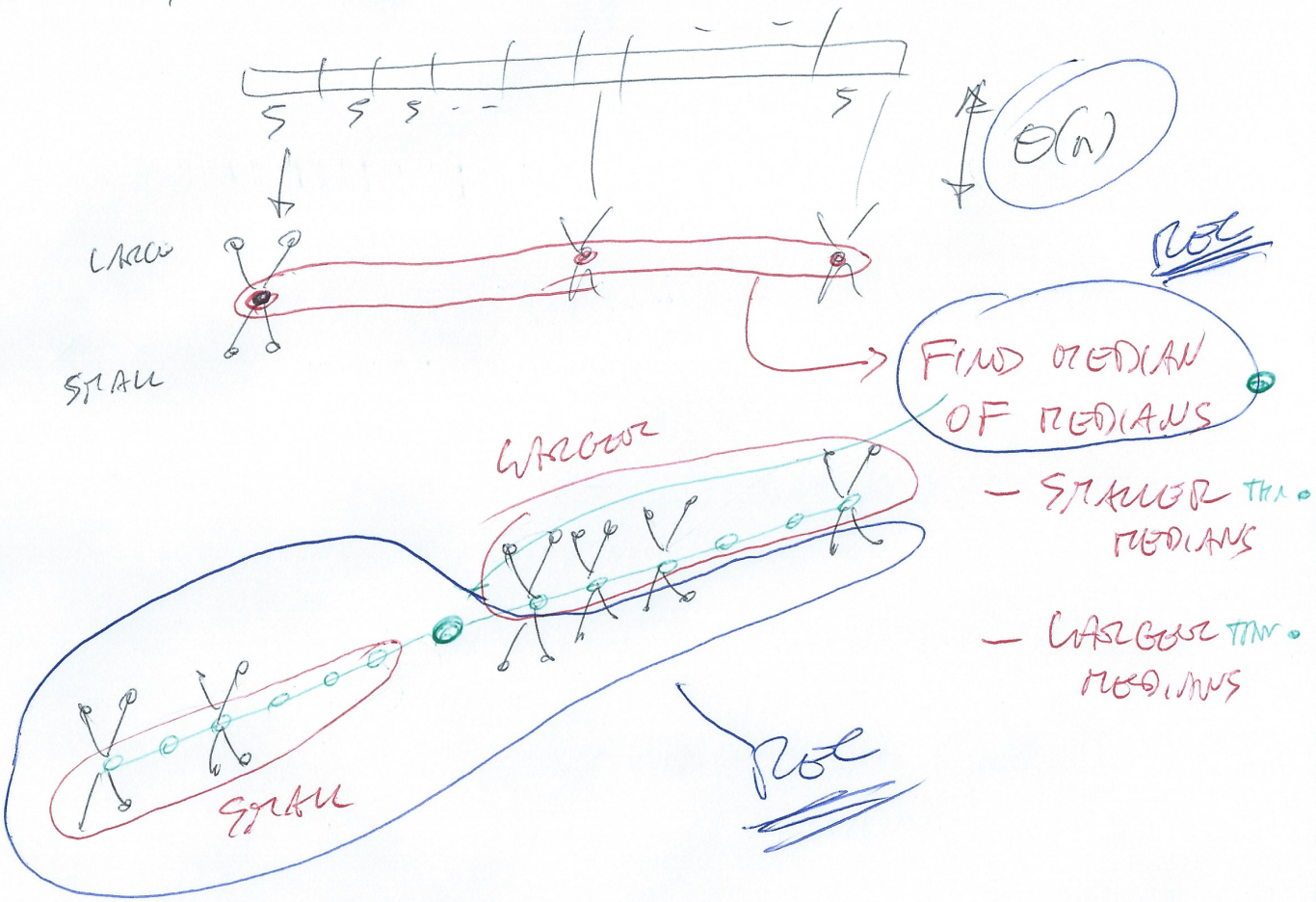
SELECTION (SEQ, START, END, K)

- BASE STEP { START = END } → K_2
  DONE
- PARTITION
  - ↳ EITHER FOUND K-th
  - ↳ RECURSE ON THE APPROPRIATE PORTION

6 SELECTION k-th elem (MEDIAN) $\Theta(n)$ WORST CASE TIME

ALWAYS OPTIMAL

[BLUM, FLOYD, PRATT, RIVEST, TARJAN]



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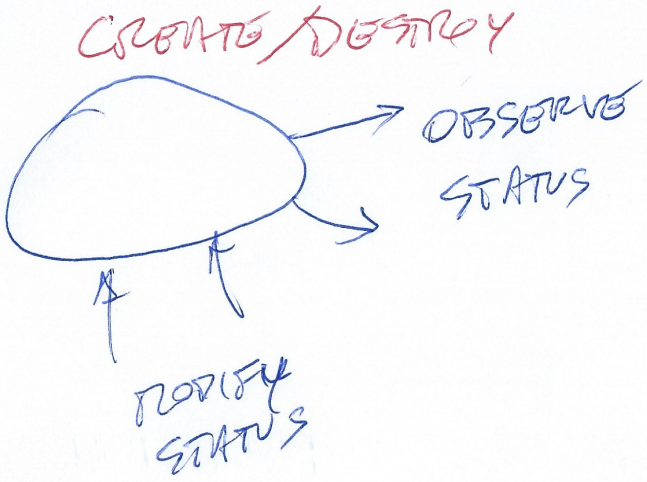
ABSTRACT DATA TYPE (ADT)



EXAMPLE

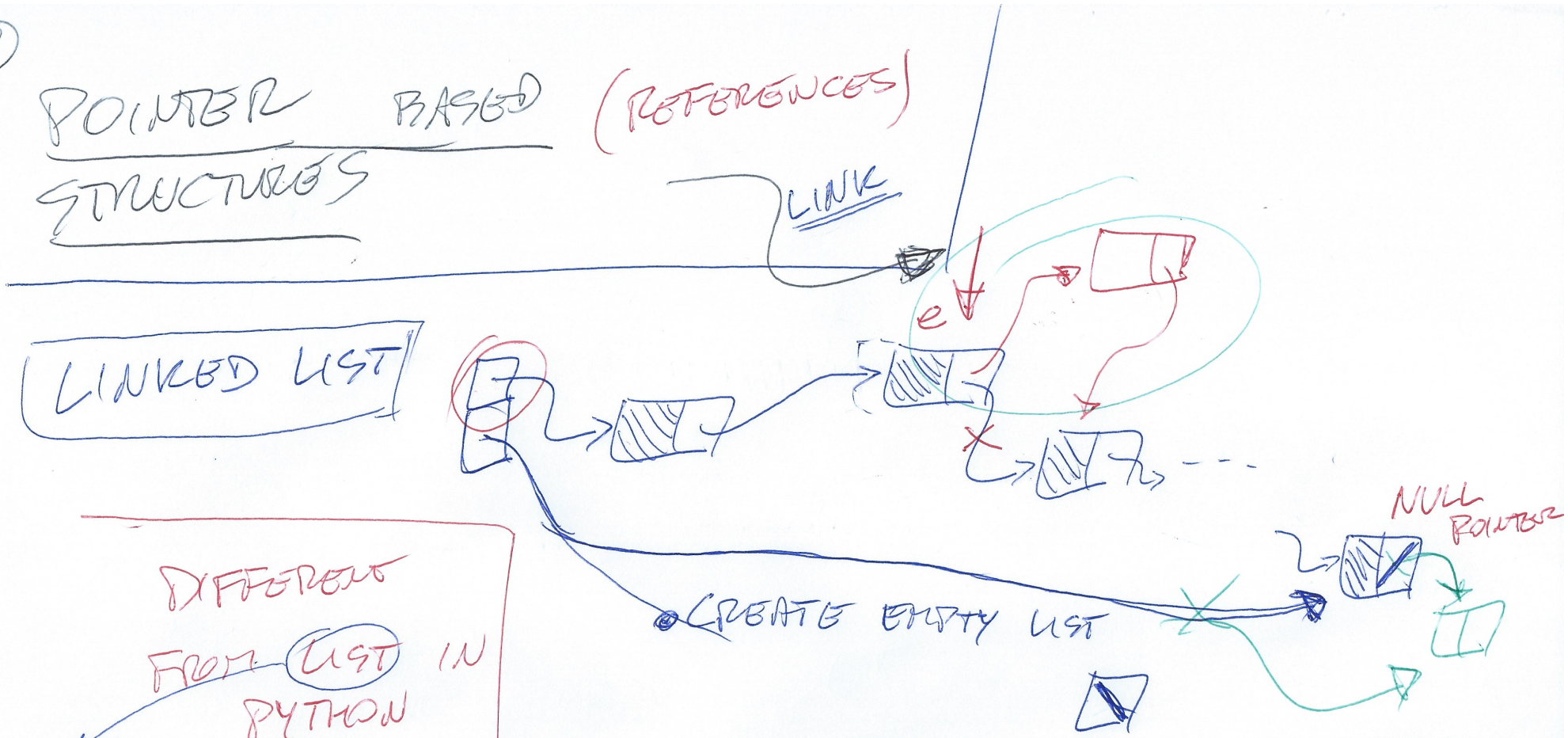
- ADD A VALUE TO
- ASK WHETHER X HAS BEEN EVER ADDED
- HOW MANY VALUES ARE THERE
- IS IT EMPTY?

SET

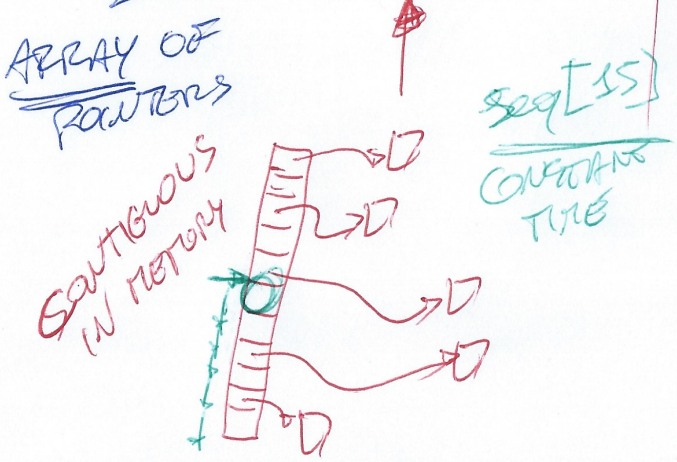
- CREATION EMPTY SET
- ASK $x \in S$ TRUE IFF OPERATION ADD(x) HAS BEEN PERFORMED BEFORE

8

POINTER BASED STRUCTURES (REFERENCES)



DIFFERENT FROM LIST IN PYTHON



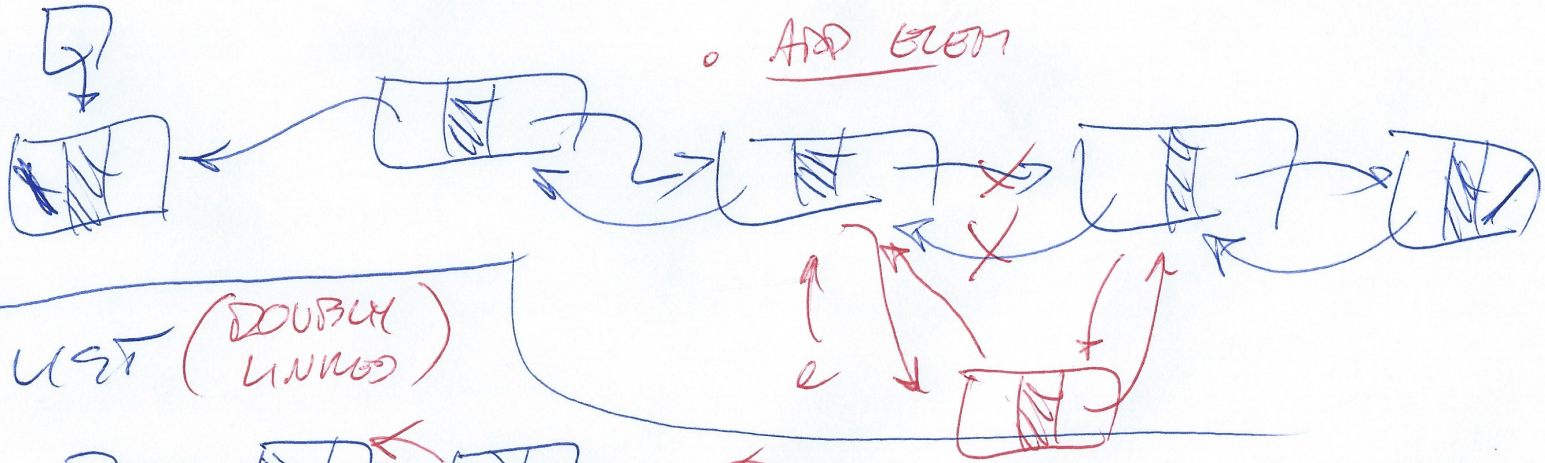
- GET FIRST ELEM. (IF EXISTS) $\Theta(1)$
- GET i-th ELEM. (if EXISTS) — SCAN LIST TO THE i-th.
- GET LENGHT.
- ADD ELEMENT < TOP, AFTER e, END

LINKED LIST:

- ADD AFTER e $\Theta(1)$ KNOWING A REF TO e
- NEXT(e) $\Theta(1)$ KNOWING e REF
- PRED(e) ??? SCAN FROM THE START !! $\Theta(n)$
- LAST

DOUBLY LINKED LIST

- PRED(e) $\rightarrow O(1)$
- ADD ELEM



CIRCULAR LIST (DOUBLY LINKED)