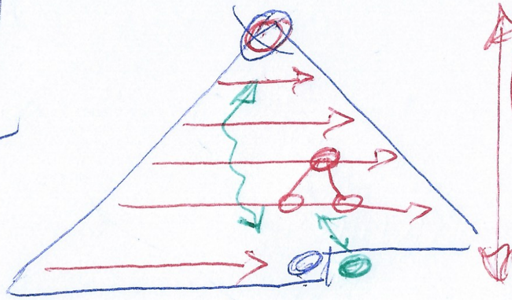
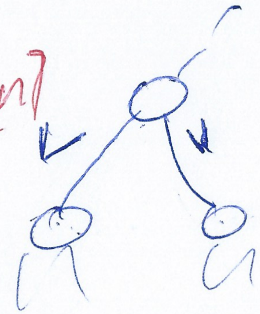
1 PRIORITY QUEUE BY HEAPS

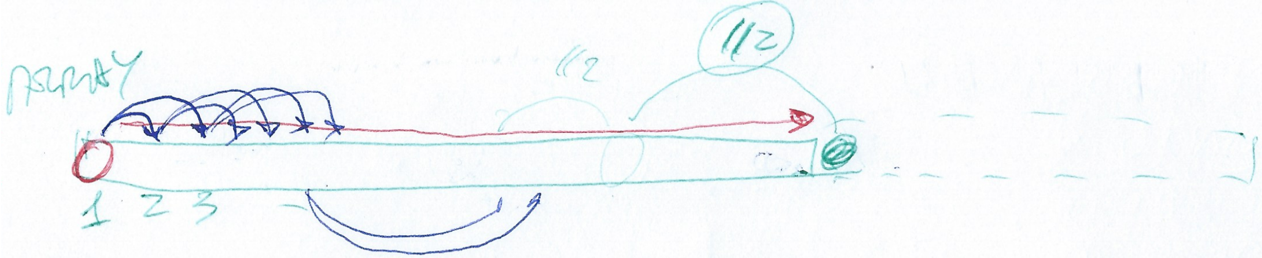
IMPLEMENT AS AN ARRAY

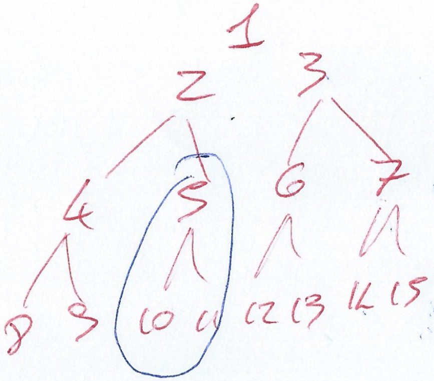


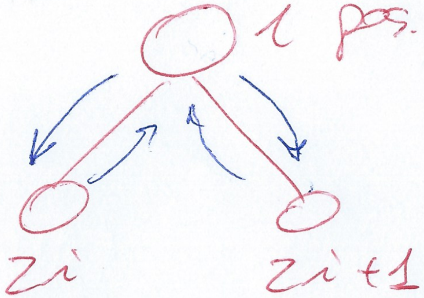


- INSERT
- DELETE MAX
- INCREASE KEY

$\Theta(\lg n)$

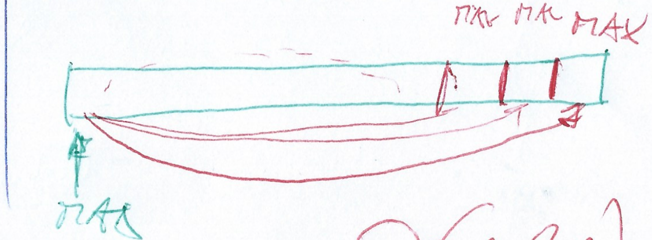






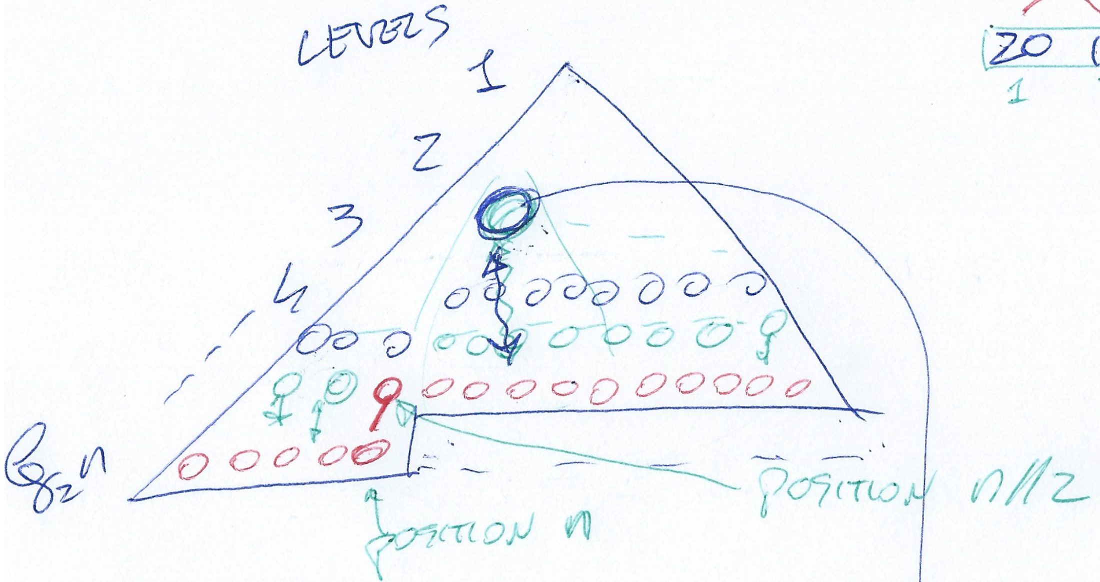
HEAP SORT (ON PLACE)

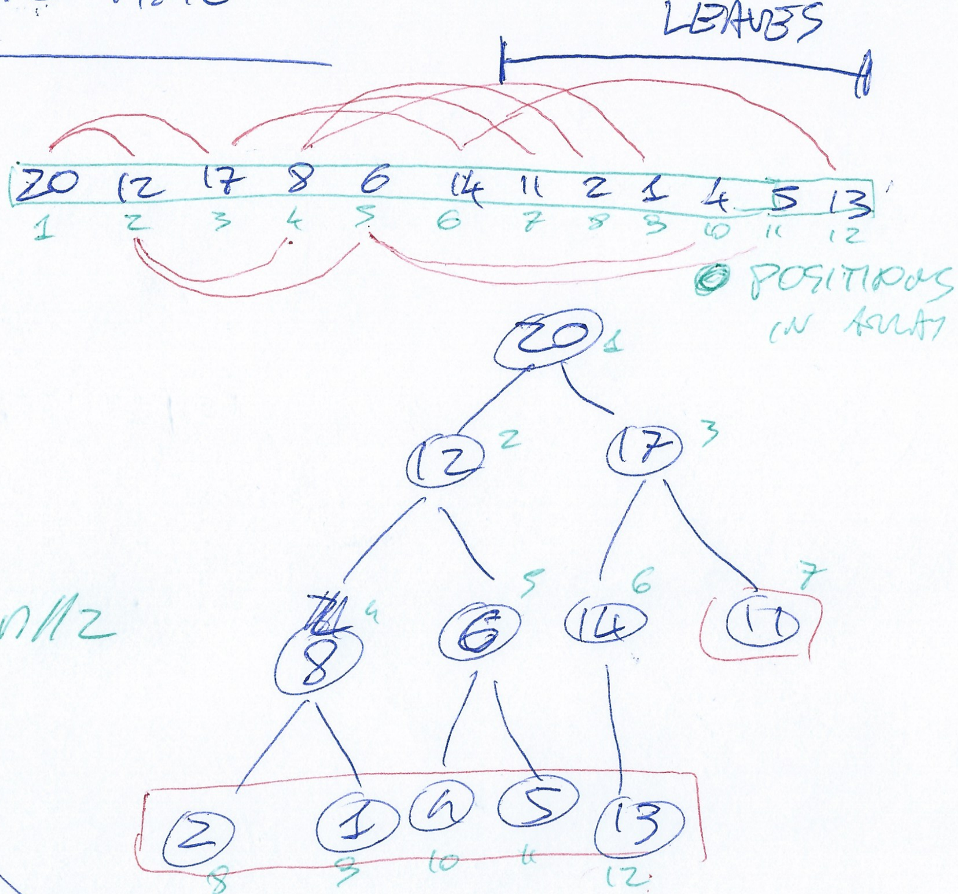
- BUILD A HEAP (n elements) $n \cdot \lg n$?
- ~~GET~~ DELETE MAXIMUM UNTIL HEAP IS EMPTY $n \cdot \lg n$ (n)

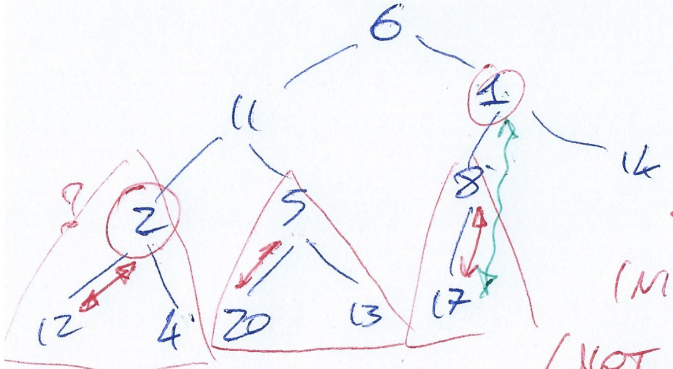


$\Theta(n \lg n)$

② HEAPIFY TAKES $\Theta(n)$ W.C. TIME







~~START~~ INITIAL ARRAY (NOT A HEAP) : BUILD A HEAP!

ELEMENTS IN LEVEL i COULD GO DOWN TO LEVEL $\rightarrow \log n$ AT MOST

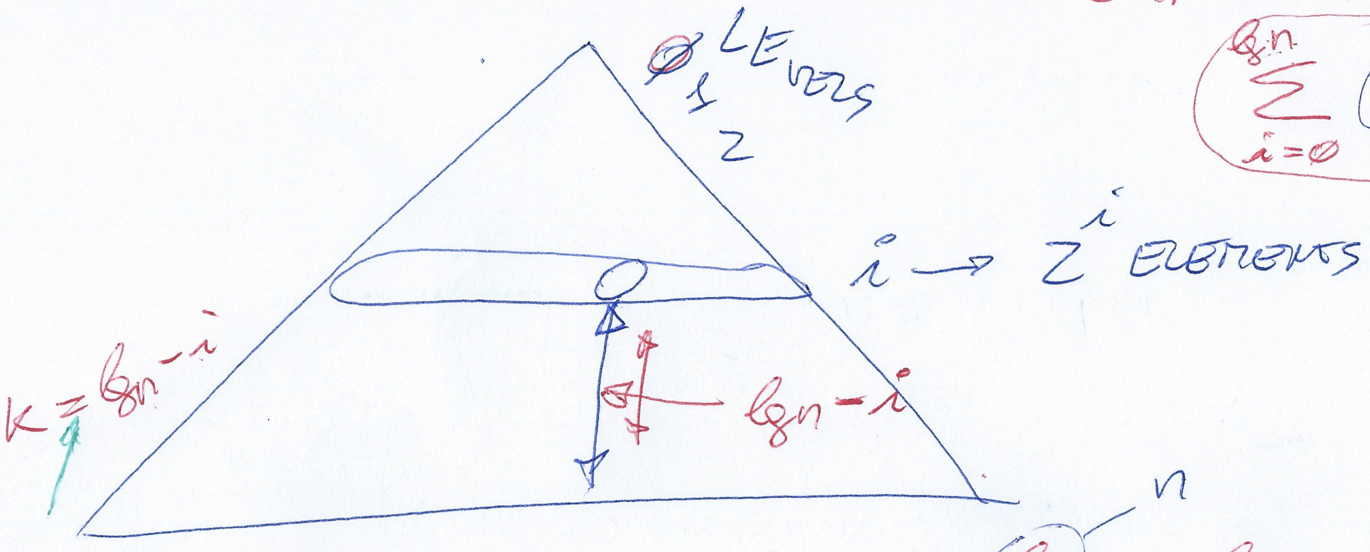
(3) HEAPIFY IN $\Theta(N)$

$n = 2^k - 1$ COMPLETE

COMPLEXITY OF HEAPIFY IS

$$\sum_{i=0}^{\lg n} 2^i \cdot (\lg n - i) \le$$

ELEMENTS LEVEL i



$$\le \sum_{k=0}^{\lg n} k \cdot 2^{\lg n - k} = \sum_{k=0}^{\lg n} \frac{k \cdot \overbrace{2^{\lg n}}^{n}}{2^k} = \sum_{k=0}^{\lg n} \frac{k}{2^k} \cdot n = n \cdot \sum_{k=0}^{\lg n} \frac{k}{2^k} <$$

$$< n \cdot \sum_{k=0}^{\infty} \frac{k}{2^k} \longrightarrow \Theta(n)$$

SMALL CONSTANT $= 2$

(2c) SHOWING THAT

$\sum_{k=0}^{\infty} \frac{k}{2^k}$ IS CONSTANT $= 2$

$\sum_{k=0}^{k} x^k$

$(x^{k+1} - 1) = (x-1)(x^k + x^{k-1} + x^{k-2} + \dots + x^2 + x + 1)$

$f(x)$

$\sum_{k=0}^{\infty} x^k = \frac{1}{1-x}$ IF $x<1$

$\frac{d(x^k)}{dx} = k x^{k-1}$

$\sum_{i=0}^{\infty (k)} x^i = \frac{x^{k+1} - 1}{x - 1}$

<1

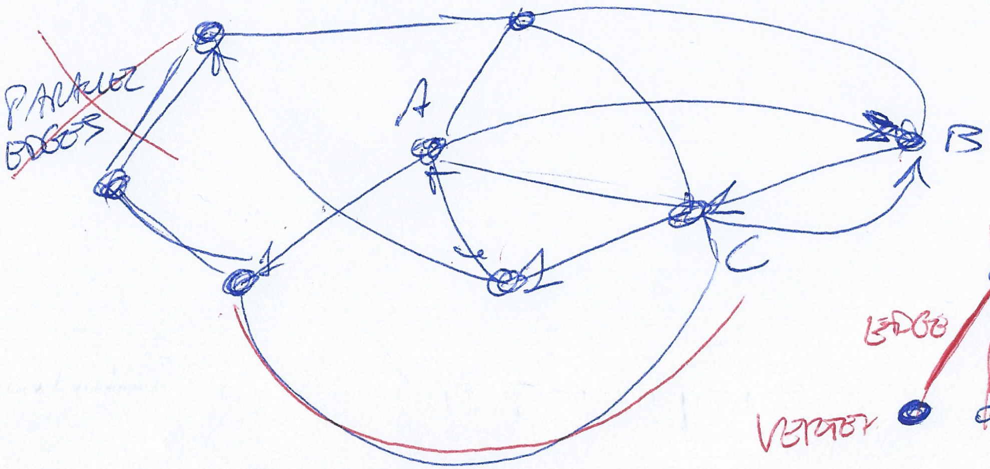
$\sum_{k=0}^{\infty} k \cdot x^k$? $= \frac{d\left(\frac{1}{1-x}\right)}{dx} \cdot x = \frac{+1}{(1-x)^2} \cdot x$

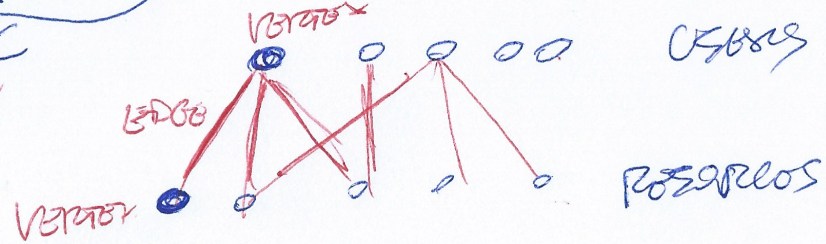
$f'(x) \cdot x$

APPLY WITH $x = \frac{1}{2}$

$= 2$

5 GRAPHS





$$G = (V, E)$$

V VERTICES : SET

$E \subseteq V^2$: SET OF PAIRS OF VERTICES

EDGES

$|V| = n$

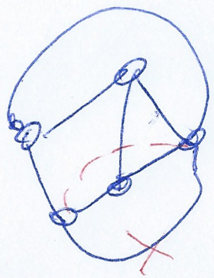
$|E| = m$

$$\emptyset \leq m \leq \binom{n}{2} \approx n^2$$

$$\frac{n(n-1)}{2}$$

DENSE GRAPH

$$m = \Theta(n^2)$$



PLANAR GRAPH

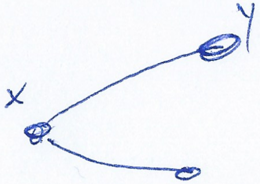
CAN BE DRAWN ON THE PLANE WITH NO INTERSECTIONS

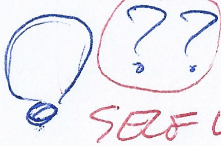
SPARSE GRAPH

$$m = \Theta(n)$$

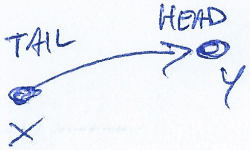
6 GRAPHS

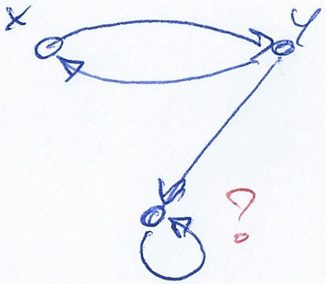
- UNDIRECTED
  EDGES

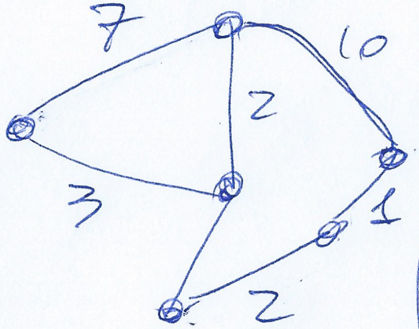




- DIRECTED
  ARCS







- UNWEIGHTED
- WEIGHTED

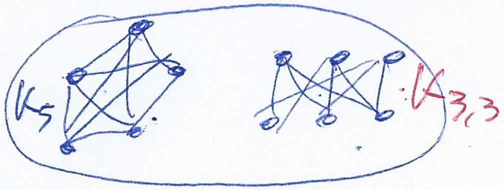
$G = (V, E, w)$

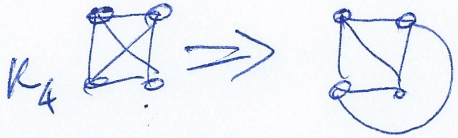
$w : E \longrightarrow \mathbb{R}^{(+)}$?

$e \in E$ $\quad w(e)$ WEIGHT OF EDGE e

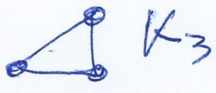
SPECIAL GRAPHS

- PLANAR ? $\Leftrightarrow$ DOES NOT CONTAIN





- CLIQUES : COMPLETE



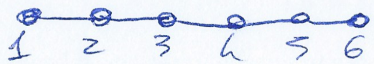
K_n

$|V| = n$

$|E| = \frac{n(n-1)}{2}$

- PATH

$$P_n = \begin{cases} V = \{v_1, v_2, v_3, \dots v_n\} \\ E = \{(v_i, v_{i+1}) : 1 \le i \le n-1\} \end{cases}$$

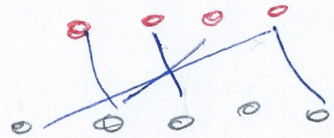


P_6

7) CYCLES

$C_n \simeq P_n \underset{\simeq}{+} (v_n, v_1)$

BIPARTITE GRAPHS



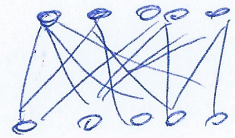
? n^2 EDGES?

$G = (V_1, V_2, E)$

PAIRS (x, y)

$E \subseteq V_1 \times V_2$

$x \in V_1$
$y \in V_2$



$\frac{n}{2}$

$\frac{n}{2}$

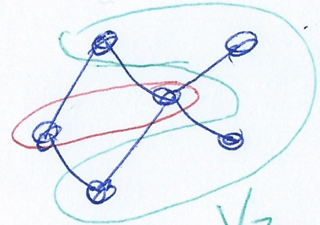
$|E| = \frac{n}{2} \cdot \frac{n}{2} \approx \Theta(n^2)$

$E = V_1 \times V_2$

COMPLETE BIPARTITE GRAPH

K_{n_1, n_2}

V_1



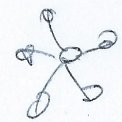
V_2

??

?

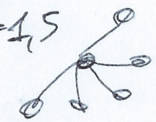
$|E| \le \binom{n}{2} = \Theta(n^2)$

$|E|$ BOUNDED?



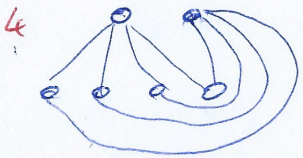
STAR

$K_{1,5}$



$K_{2,4}$

PLANAR?

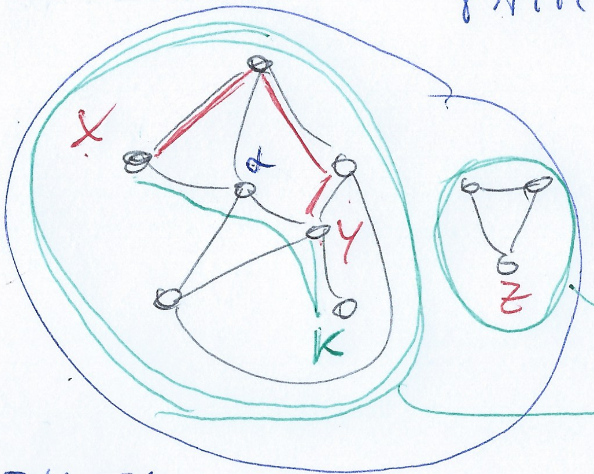


8 CONNECTIVITY

- $G = (V, E)$, $x, y \in V$

G IS CONNECTED
IFF $\forall x, y \in V$
G CONTAINS PATH $(x \Leftrightarrow y)$

x, y ARE REACHABLE $:$ G CONTAINS A PATH $x = v_1, v_2, v_3, \ldots v_n = y$



x, y REACHABLE

y, z NOT REACHABLE

G IS SPLIT INTO CONNECTED COMPONENTS

REACHABILITY

- SYMMETRIC $(x \rightsquigarrow y \Leftrightarrow y \rightsquigarrow x)$
- TRANSITIVE $x \rightsquigarrow y,\ y \rightsquigarrow b$
$$\Downarrow$$
$$x \rightsquigarrow b$$

x, k REACHABLE x, α, y, k
k, y REACHABLE k, y

x, α, y, k, y (NOT SIMPLE PATH)